# Mathematical Pattern Poetry 

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#### Abstract

Pattern poetry is an art form that combines literary and visual elements to produce poems in which the text and the shape of the poem work together as an aesthetic whole. To fully appreciate a pattern poem one needs to see it as well as hear it. Although varied forms of this art existed throughout history and across cultures, it is only in the $19^{\text {th }}$ century that pattern poetry became a recognized genre of poetry. This genre is further enriched by the increasing graphing capabilities of modern computers that add a new dimension to the hand-drawn or typewriter-produced pattern poems of the past. Mathematical pattern poetry-pattern poetry possessing a substantial mathematical component-includes poems whose shape or content involve geometric figures, mathematical curves, or other mathematical notions and symbols. This article provides a brief overview of modern mathematical pattern poetry, including selected references for the use of such poetry in the mathematics classroom. The concluding remarks touch lightly on the poetry forms that sprang from this genre: visual poetry and electronic poetry.


## Plane Figures and Curves

One of the earliest known pattern poems, The Sator Square [28], has a mathematical shape. It is a $5 \times 5$ letter-square containing the Latin palindrome SATOR AREPO TENET OPERA ROTAS. Representations of The Sator Square were found in numerous European archeological sites, the earliest going back to the ruins of Pompeii, which was buried under volcanic ash in 79 AD. A rough translation is: "The farmer Arepo uses his plough as his form of work." The significance of the square is mysterious. Speculations connect it to various letter rearrangements which lend it an early Christian religious association. Later square
 poems were constructed as labyrinths or involved other internal puzzle-patterns that were religious in nature [15]. An interesting example is the $16^{\text {th }}$ century $10 \times 10$ syllable-square from Sundry Christian Passions by Henry Lok, Square Poem in Honor of Elizabeth I [11, 34].

Below is a modern square poem by JoAnne Growney, which appears in [12] for Earth Day, April 21, 2011. It is an $8 \times 8$ syllable-square poem concerned with the depletion of one of our most important resources-water. Mathematical in both shape and content, the poem describes an exponential-rate consumption of water. But one does not need to understand the mathematics of exponential growth in order to fully absorb the message of this poem. Substituting "syllable" for "quart" and doing the calculation described in the poem, one can practically see the water level dropping as the poem disappears syllable by syllable: On the first day 1 syllable is gulped; on the second day 2 syllables are gulped, making the total number of gulped syllables after two days $1+2=3$; the total number of gulped syllables after 3 days is $1+2+2^{2}=7$. How long before no water is left? Gone in a week.... In addition to aesthetic value this poem is particularly suited for use in an algebra class to enhance the
teaching of the topic of exponential growth and decay. Poetry projects are used in the mathematics classroom for their power to engage attention and enhance memory. Careful project construction often results in additional pedagogical benefits, such as better integration of material and easier transition to its applications. Ideas on how to construct poetry projects and a survey on the use of poetry in college mathematics classes may be found in [7, 8, 9,10].

To the right is a pattern poem by Li C. Tien, Right Triangle [33]. It involves squares and triangles, although these shapes are not made up by counting syllables. The image brings to mind the various proofs of Pythagoras Theorem [2] and, like the previous poem, Right Triangle could be used to enhance the teaching of this topic in an algebra class. Squares and triangles are not the only two dimensional figures used to construct pattern poems. Gerald Lynton Kaufman's Geometric Verse: Poetry Forms in Mathematics Written Mostly for Fanatics [18] contains pattern poems involving rectangles, rhombuses, trapezoids, polygons, circles, ellipses, and much more. Other sources of pattern poetry involving two dimensional figures are $[1,4,11,14,15,27]$.

We conclude our discussion of pattern poems whose shape involves planar figures with two poems involving curves. Below, on the right, is: Riddle in the Shape of an Octagon, by the Mexican Nobel laureate poet Octavio Paz [24]. It involves 8 line segments radiating from a center-an image that can be viewed as a division of an octagon. Following the poet's instructions one can reorder and place the 8 statements appearing under the geometric figure in the octagon's "slices" to read an enchanting poem. Try it! Lines also appear in Eugene Guillevic's poems: Line, Parallels,

## Right Triangle

by Li C. Tien
 and Perpendicular [14]. On the left is Gerald Lynton Kaufman's Spiralay [18], whose shape is a spiral. Spirals are graphs of certain functions in polar coordinates, such as, for example, the logarithmic spiral $r=a e^{b \theta}$ [1]. Spirals as shapes for poems also appear in The Derivative, by Betsy Adams [26], and Spiral by Eugene Guillevic [14]. The latter also includes other curve patterns such as parabolas, cycloids, sinusoids, and hyperbolas.

## Spiralay <br> by Gerald Lynton Kaufman



Riddle in the Shape of an Octagon by Octavio Paz
Place each one of the 8 phrases on each of the 8 lines so that, read from
1 to 8 , they form parallel statements.
1 You at the center shape of An octacon
2 The sun's knife
3 Divides this octagon
4 Eyes nose hands tongue ears
5 East and West North and South
6 Cuts this bread
7 The abyss is at the center
8 See smell touch taste hear

## Solids，Möbius Strips，and Fractals

In a cylindrical pattern poem called Iambi－Cylinder，Gerald Lynton Kaufman［18］writes：

> iambic cylinders are seldom found because their music goes around and round so publishers who read them never know what's printed on the side that does not show

In other words，poets who try to write a pattern poem with a three dimensional shape，or a two dimensional shape that does not lie flat on the plane，encounter the same difficulties mathematicians have when trying to draw three dimensional figures on a flat piece of paper．It is interesting to note that there is more than one solution this problem．

The lovely little poem to the left，The Truncated Cone by

Truncated Cone
by Eugene Guillevic


Eugene Guillevic［14］，provides one of the simplest solutions－ place the poem and the figure representing its shape separately－ but in close proximity．Effective，but perhaps not satisfying the impulse to integrate the text and the form of the poem．

A different solution is offered in Tyehimba Jess＇poem，The Bert William／George Walker Paradox［17］．The poem is displayed on the page in two equal size columns：the left column for Bert Williams＇words，and the right column for George Walker＇s words． The reader is advised to cut the poem＇s text out of the page and roll，fold，and glue as necessary to read the poem on cylinders or on a Möbius strip．We reproduce a portion of the poem and summarize the reading instructions below．As background to the poem the poet provides the following introduction：＂Bert Williams and George Walker were an African American comedy duo who， through extraordinary skill and insight，were able to depict subtlety，humanity and intelligence within the confines of the only site black comedic actors could work at the turn of the $19^{\text {th }}$ to $20^{\text {th }}$ century－the minstrel show．Bert Williams（1874－1922）became known as＂The Jonah Man＂and was known for his trademark song＂Nobody．＂George Walker（1873－ 1911）was known for his dancing skills and played the straight man in their act．＂

## From：The Bert William／George Walker Paradox

by Tyehimba Jess

Sing like me，Jonah in the charcoal hold of the whale－ be smiling till the crowd starts to cry：this nobody
born to riddle all＇bout race．A sure bet nobody might be saying＂look at that handsome man！＂Nobody
wondering bout dark＇s pathos．Believe the human being buried＇neath my ash．Don＇t claim that nobody
holding secret smiles for another man＇s plight．A shame
knuckled under humor．Who escapes it？Nobody

Doing justice to my Juba jig．See how I dance？ just might be playing you for a fool．You see this face got them laughing loud like me．You think they mock my face？ Can＇t help but see themselves beneath my blacked up face－
being swept up in my diamond stud strut：Bless those that getting touched from inside out．Every wide eyed face
in the crowd all gut－bent with laughter－they know that they will be baptized clean．I want to make it so－each face

Below is a summary of the instructions given by the poet for turning the poem into a pattern poem:

1. Cut the poem's text along the four dotted lines.
2. To read the poem on a horizontal cylinder roll the poem's text and glue the top and bottom dotted lines.
3. To read the poem on a vertical cylinder roll the poem's text and glue the two side dotted lines.
4. To read the poem on a band, fold the poem's text along the line separating the columns. Roll the resulting half text and glue top to bottom. You may now read the poem on the circular band from the inside as well as the outside.
5. To read the poem on a Möbius strip, after folding the poem's text down the middle as in step 4 , give it a half twist before you glue the ends. The poem can now be read along a two dimensional surface with only one side-a Möbius strip. To the right is the poem placed on a Möbius strip.

Another solution for presenting a three dimensional pattern poem on the flat page is to provide an illusion of three dimensionality. This can be achieved through various techniques. Gerald Lynton Kaufman [18], who was an architect, achieved this
 effect through the use of perspective when drawing the shape and the letters of the poems in a way similar to its use for drawing architectural diagrams. A particularly ingenious example is his poem Cubicouplets [18], which we cannot include here because of space limitations. The poem manages to present the words written on both the seen and the "unseen" side of a cube.

My poem to the left, Do You Believe in Fairy Tales?, employs
Do You Believe in Fairy Tales? by Sarah Glaz
 a different technique to create an illusion of a three dimensional spherical object. The object-a planet or a moon-is evoked through typography. The poem's lines are arranged in a way that their tips draw a parabola and the resulting image displays characteristics that are universally associated with moon images. The reader's imagination does the rest. Caleb Emmons poem, Seeing Pine Trees [6], is another mathematical pattern poem employing such technique for the visualization of a non-mathematical three dimensional object-an evergreen.

We conclude this section with another pattern poem that employs illusion for its presentation. The poem, The Cantor Dust - a Fractal Poem, by Rodrigo Siqueira [30], which appears on the next page, uses the visual structure of the Cantor Set to send a poetic and mathematical message. The Cantor Set is constructed by repeatedly deleting the middle third from each interval of a growing collection of intervals. The construction starts with the deletion of the middle third from the interval $[0,1]$. Then, the middle third from each of the two remaining intervals is deleted. Next follows the deletion of the middle third from each of the four remaining intervals, and so on. The process never terminates. Although this process does not yield a three dimensional object it is difficult to depict it in any dimension because it never ends. Siqueira's poem uses shape, size, shading, empty spaces and the reader's imagination to fill in the gap between the finite and the infinite. The sources cited in this section and [26] contain many other interesting examples of three dimensional mathematical pattern poems.


## Beyond Geometry and Topology

In addition to geometric and topological patterns, mathematics is rich in symbols, notation, and various possibilities of manipulating entities that can be used as patterns for poems. Given the place numbers hold in mathematics, it is perhaps surprising that there is no abundance of poems utilizing the shapes of numbers. Still, such poems exist. Below is a small fragment from the book length poem Midpoint by John Updike [35], where numbers paint a more vivid image from one's childhood than any number of words can do. A poem shaped like number 1 by Thomas

From: Midpoint: The Play of Memory by John Updike
 Eisele appears in [5]. Other poems utilizing numbers appear in [26].

The symbol signifying the "end" of the number line, the horizontal eight-infinity-appears either on its own or in correlation with other numbers in several poems of [26] and [18]. Below is Bernhard Frank's Infinity [26], a poem that comments on its own shape as well as on a certain difficult human condition.

Infinity
by Bernhard Frank


The last group of mathematical pattern poems we will discuss in this article involves symbols and words that denote mathematical objects or operations, in a variety of surprising and delightful ways. Below, to the left, is a small fragment from Lois Zukofsky $A$ series poems [36]. It employs the mathematical symbol for an integral to give a concise explanation of his poetics as the integration of speech and music. The integral sign explains visually what the words below it, using the mathematical language of upper and lower limits of integration, do verbally. The combination is aesthetically pleasing in the way a bilingual version of an original poem and its good translation is pleasing for a reader fluent in both languages. Similarly, Bob Grumman's Mathemaku No. 10 [11, 13] employs mathematical notation to extend the capability of language and express the poem's intent in a more compact, minimalist, way. Grumman went one step further than Zukofsky and omitted some of the words needed to fully explain the mathematical iconography. This adds an intriguing layer of ambiguity, but in the same time sacrifices the opportunity of making music through speech. Mathemaku No. 10 is one of the early mathemakus, mathematical haikus, created by Bob Grumman. His mathemakus evolved over time to become more visual by including more symbols and images and less words. This trend from pattern poetry to what is called nowadays visual poetry, is apparent in the works of other poets who create visual poetry-a subject we will return to in the concluding remarks. The third poem in this grouping, using an even smaller amount of words, is Kaz Maslanka's, Sacrifice and Bliss [11, 21]. In addition to mathematical notation this poem involves a mathematical notion which requires the reader to actually "do mathematics" on the words in order to understand the poem. Specifically, as the denominator of the fraction, the ego, shrinks to 0 , the fraction grows larger and larger, till-at the limit-it becomes infinite. In other words, love's strength is inversely proportional to the size of the ego. Maslanka's poetry [21], which he names equation poetry, involves several other mathematical operations. He also embeds many of his poems in computer enhanced visual environments that intensify their visual impact.

From: A 12
by Louis Zukofsky


## Sacrifice and Bliss

by Kaz Maslanka

$$
\text { love }=\lim _{\operatorname{cg} \rightarrow 0} \frac{1}{\text { ego }}
$$

Mathemaku No. 10
by Bob Grumman

$$
\text { Mathemaku No. } 10
$$

poetry $\longdiv { \frac { \square } { \text { existence } } }$
somewhere, minutely, a widening existence

The last poem in this group, 26 Points to Specify [25], by the French surrealist poet Benjamin Péret, appearing on the next page, uses mathematical symbols to extend the capability of language in a different way from the three poems mentioned above. The mathematical formulas at the end of each of the poem's 26 lines complement what is said by what is unsayable-or by that which is better left unsaid. If I may venture an interpretation, the ever increasing complexity of the formulas as the poem progresses backwards from death to birth

## From: 26 Points to Specify

by Benjamin Péret
My life will end by $a$
I am $b-a$
I ask for $c b-a$
I consider the religious holidays $\frac{d}{c b-a}$
My prediction for the future $\frac{d e}{c b-a}$

My fortune $\left(\frac{\frac{m}{n}\left(\sqrt[g]{\frac{d e}{(c b-a) f}}\right)^{j}+k l+o}{\sqrt[t]{(p q+r) s}}-u v-w\right)^{x}-y$
The date of my birth $\left(\frac{\frac{m}{n}\left(\frac{g}{\frac{d e}{(c b-a) f}}\right)^{j}+k l+o}{\sqrt[t]{(p q+r) s}}-u v-w\right)^{x}-\frac{y}{z}$
is meant to express the richness of the gifts with which we are born and their gradual diminishment as we age.

Several other poets employ mathematical symbols and formulas as a partial pattern embedded in a traditional poem in order to extend the capability of language. The reasons for using this kind of pattern and its meaning in any particular poem are complex, and vary from poem to poem. Examples of poems using this technique include: Ray Bobo's Give Me an Epsilon and I will Treat It Well [11], Lewis Carroll's Yet What Are All Such Gaieties [3], Velimir Khlebnikov's Order from the Presidents of Planet Earth [19], H. K. Norla, Of Xs and of Ys [23], Ed Seikota's Borderline-a Fractal Poem [11, 29], C.K. Stead's Walking Westward [11], Stephanie Strickland's Gibbous Statement [32], and my poem, The Enigmatic Number e [8].

## Concluding Remarks

Pattern poetry is a well-established literary term with a significant history. In addition to Lok's $16^{\text {th }}$ century square poem mentioned in the first section, there are other classical pattern poems authored by, for example, Simmias of Rhodes (c. 325 BC) and George Herbert (1593-1633) [15, 34], possessing hidden mathematical dimensions. More recent, the 1897 poem: A Dice Throw by Stéphane Mallarmé [20], does as well. Due to space restrictions, we could not include an exploration of these intriguing mathematical aspects. Neither could we discuss the multitude of mathematical patterns generated by OULIPO (Ouvroir de Littérature Potentielle)-the literary movement founded in France by Raymond Queneau in 1960 [22]. OULIPO's patterns, among them the Möbius strip pattern of Jess's poem appearing in the second section, are used to construct literature to this day. The division of mathematical pattern poems into the three groups corresponding to this article's three sections, is a first step towards a classification of this poetry. Further refinements would have to take these omissions into account.

Pattern poetry is also called shape poetry, concrete poetry, or visual poetry. As the small sample presented in this article shows mathematical pattern poetry covers a wide range of possibilities, not all fitting comfortably under the umbrella of any of the names. The name "shape poetry" does not fit well poems whose shape is not geometric or topological, and the name "concrete poetry" does not fit well poems that involve the more abstract-the very non-concrete-symbols of mathematics. "Visual poetry" is a relatively new term which is better reserved for a form of poetry that evolved from pattern poetry. The term "visual poetry" describes best poetry which places more emphasis on the visual aspect of the poems than on their word-induced musicality. Good sources to learn more about this form of poetry are Geof Huth's article [16] and Kaz Maslanka's blog [21]. I picked the term "pattern poetry," as it seems to describe best the kind of poems selected for this article-poems for which literary musicality is as important as visual pattern.

In addition, it is problematic to call a poem "a mathematical poem." Maslanka's blog [21] presents a friendly, but heated, debate on what exactly constitutes mathematical poetry. To read the debate search the site using the words "Types of Mathematical Poetry." My choice is to be inclusive and call a poem mathematical if it has a significant mathematical component of any kind.

Visual poetry is not the only poetry form that evolved from pattern poetry. With the increasing capabilities of computers, it was only a matter of time, before artists introduced animation, interactivity, and even sound into their work. Visual or traditional poetry with interactive, animated, or sound elements is computer code driven. This kind of poetry is called electronic poetry. A good source to learn more about electronic poetry is an article by Stephanie Strickland [31].

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